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Porat et al.

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(54) **CLOSED FORM SINGULAR VALUE DECOMPOSITION**

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(Continued)

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H04L 5/00	(2006.01)
H04L 25/03	(2006.01)

(52) **U.S. Cl.**

CPC ***H04B 7/0626*** (2013.01); ***H04B 7/0417*** (2013.01); *H04L 5/0023* (2013.01); *H04L 25/03343* (2013.01)

(58) **Field of Classification Search**

USPC 375/295, 316, 219, 221, 259, 260, 267, 375/285, 296, 299, 346, 347, 349

See application file for complete search history.

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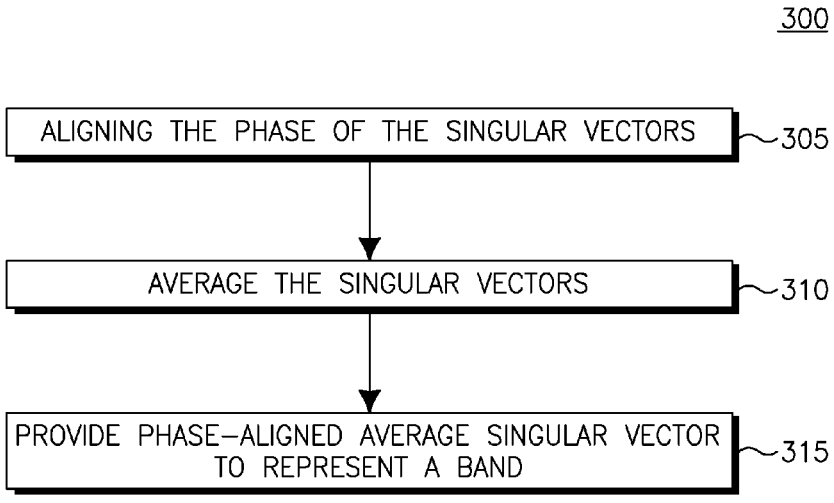
(74) *Attorney, Agent, or Firm* — Volpe and Koenig, P.C.

(57) **ABSTRACT**

The subject matter disclosed herein provides methods and apparatus for determining a singular value decomposition, providing feedback from a client station to a base station, and closed loop operation of a wireless system implementing multiple-input multiple-output (MIMO). The method may include determining one or more singular vectors using a closed form singular value decomposition. The one or more determined singular vectors may be provided to a precoder at the base station as feedback. The method may include aligning a phase of one or more singular vectors. The method may also include determining, at a client station, a plurality of singular vectors for channels used in a MIMO transmission from a base station to a client station. The client station may provide an indication to the base station regarding whether to use a singular value decomposition or a uniform channel decomposition. Related systems, apparatus, methods, and/or articles are also described.

16 Claims, 7 Drawing Sheets



Related U.S. Application Data

No. PCT/US2009/049852, filed on Jul. 7, 2009, and a continuation of application No. PCT/US2009/049853, filed on Jul. 7, 2009.

(60) Provisional application No. 61/078,766, filed on Jul. 7, 2008, provisional application No. 61/078,765, filed on Jul. 7, 2008, provisional application No. 61/078,767, filed on Jul. 7, 2008.

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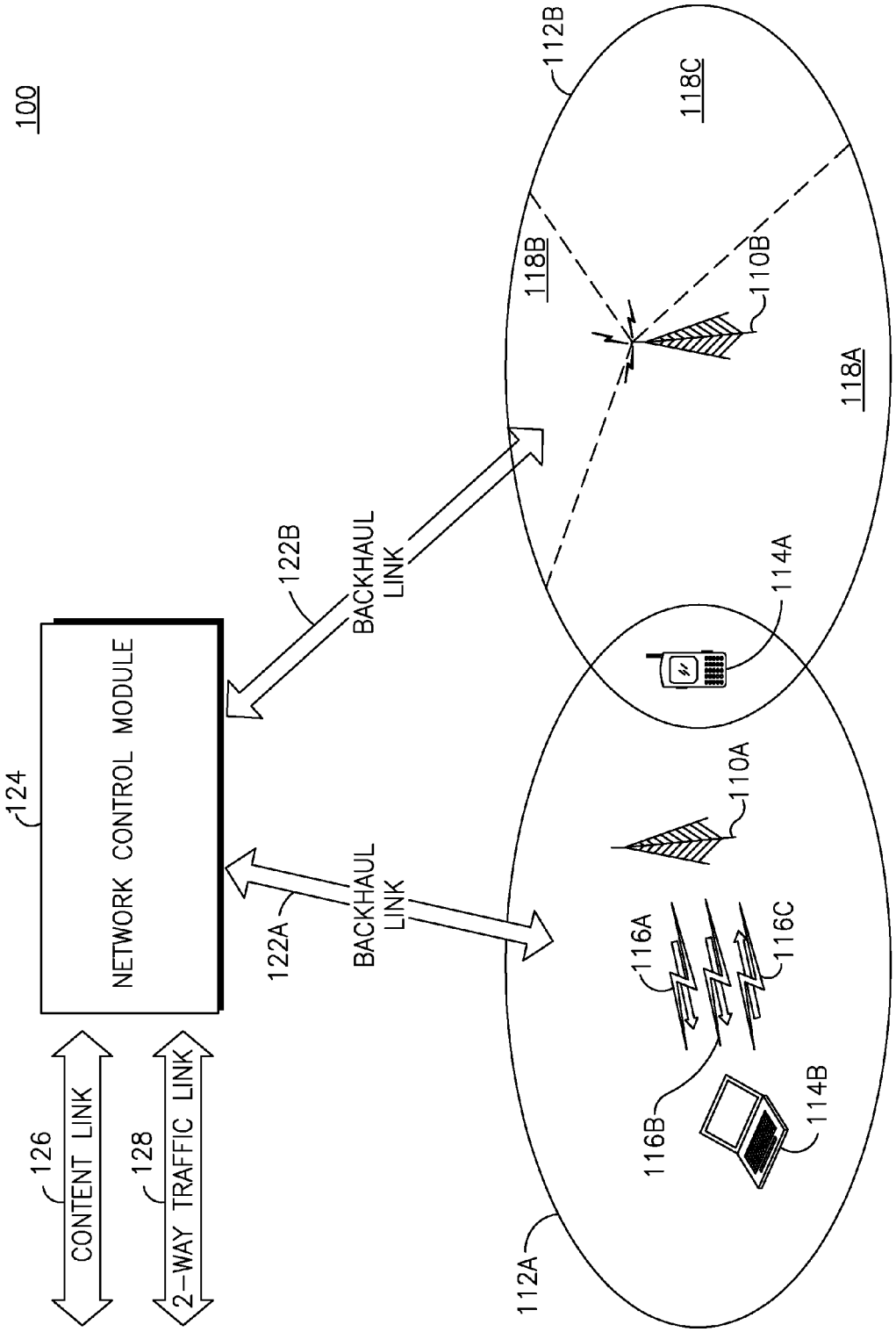
100
112B
118C
118B
110B
118A
114A
110A
116A
116C
116B
114B
112A
122B
BACKHAUL LINK
122A
BACKHAUL LINK
124
NETWORK CONTROL MODULE
126
CONTENT LINK
128
2-WAY TRAFFIC LINK

FIG. 1

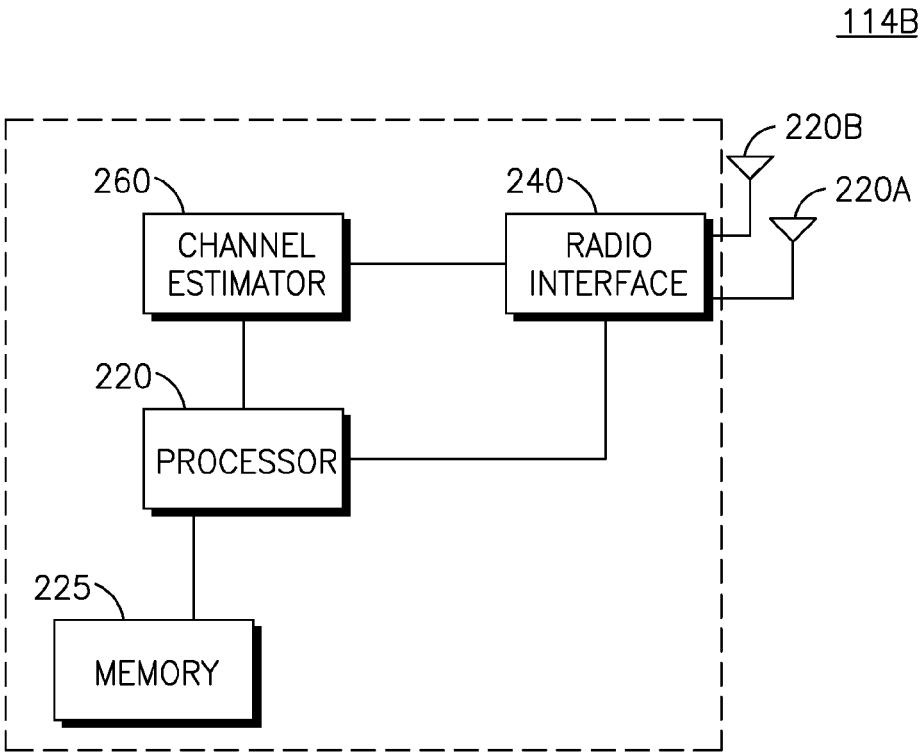
114B
220B
220A
260
CHANNEL ESTIMATOR
240
RADIO INTERFACE
220
PROCESSOR
225
MEMORY

FIG. 2

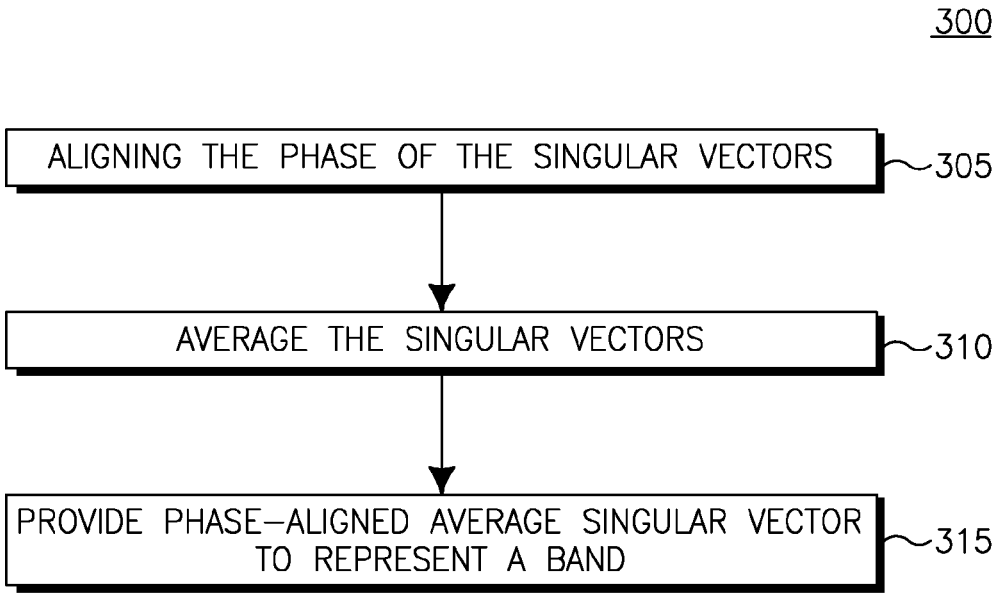
300
ALIGNING THE PHASE OF THE SINGULAR VECTORS
305
AVERAGE THE SINGULAR VECTORS
310
PROVIDE PHASE-ALIGNED AVERAGE SINGULAR VECTOR TO REPRESENT A BAND
315

FIG. 3

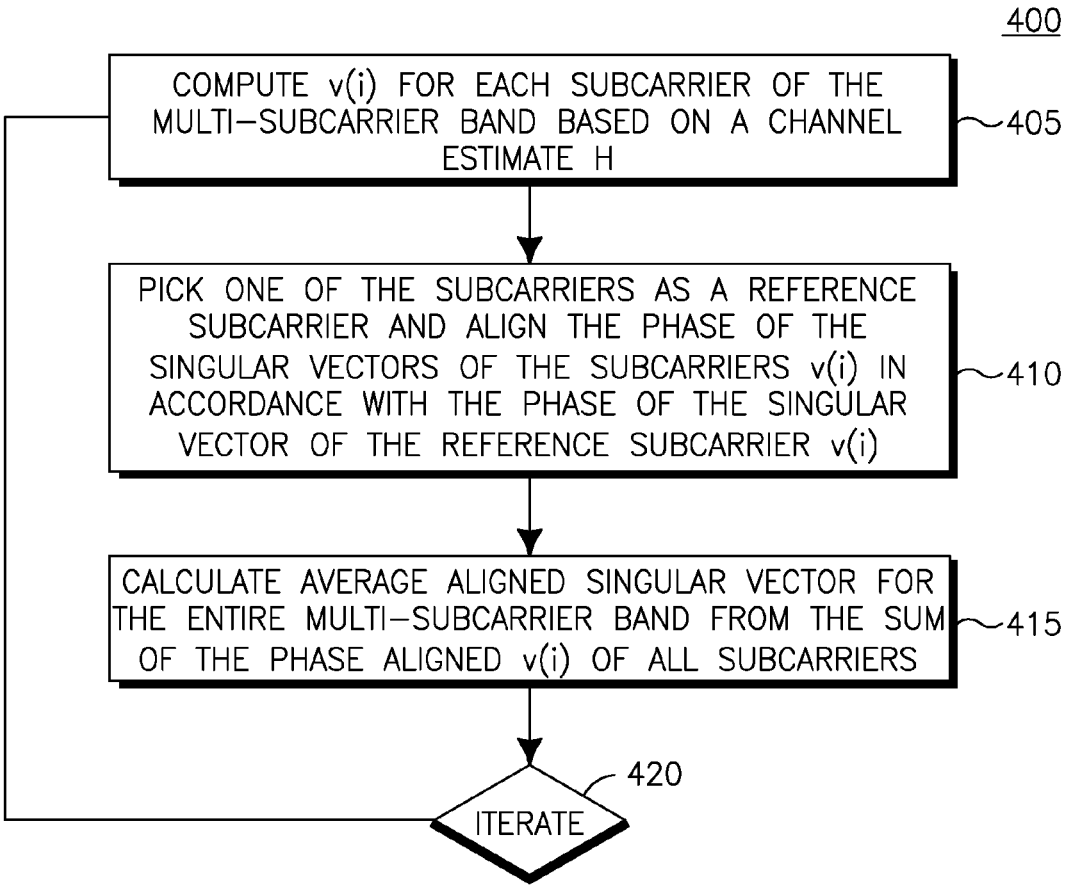
400
COMPUTE v(i) FOR EACH SUBCARRIER OF THE MULTI-SUBCARRIER BAND BASED ON A CHANNEL ESTIMATE H
405
PICK ONE OF THE SUBCARRIERS AS A REFERENCE SUBCARRIER AND ALIGN THE PHASE OF THE SINGULAR VECTORS OF THE SUBCARRIERS v(i) IN ACCORDANCE WITH THE PHASE OF THE SINGULAR VECTOR OF THE REFERENCE SUBCARRIER v(i)
410
CALCULATE AVERAGE ALIGNED SINGULAR VECTOR FOR THE ENTIRE MULTI-SUBCARRIER BAND FROM THE SUM OF THE PHASE ALIGNED v(i) OF ALL SUBCARRIERS
415
420
ITERATE

FIG. 4

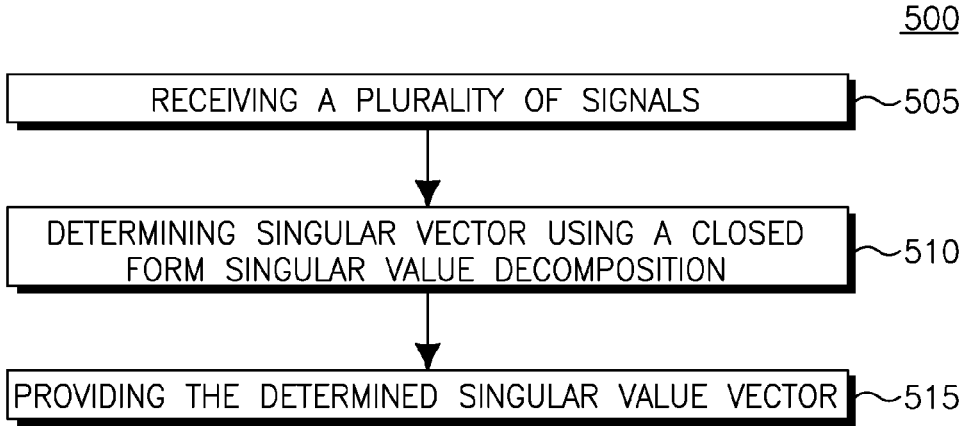
500
RECEIVING A PLURALITY OF SIGNALS
505
DETERMINING SINGULAR VECTOR USING A CLOSED FORM SINGULAR VALUE DECOMPOSITION
510
PROVIDING THE DETERMINED SINGULAR VALUE VECTOR
515

FIG. 5

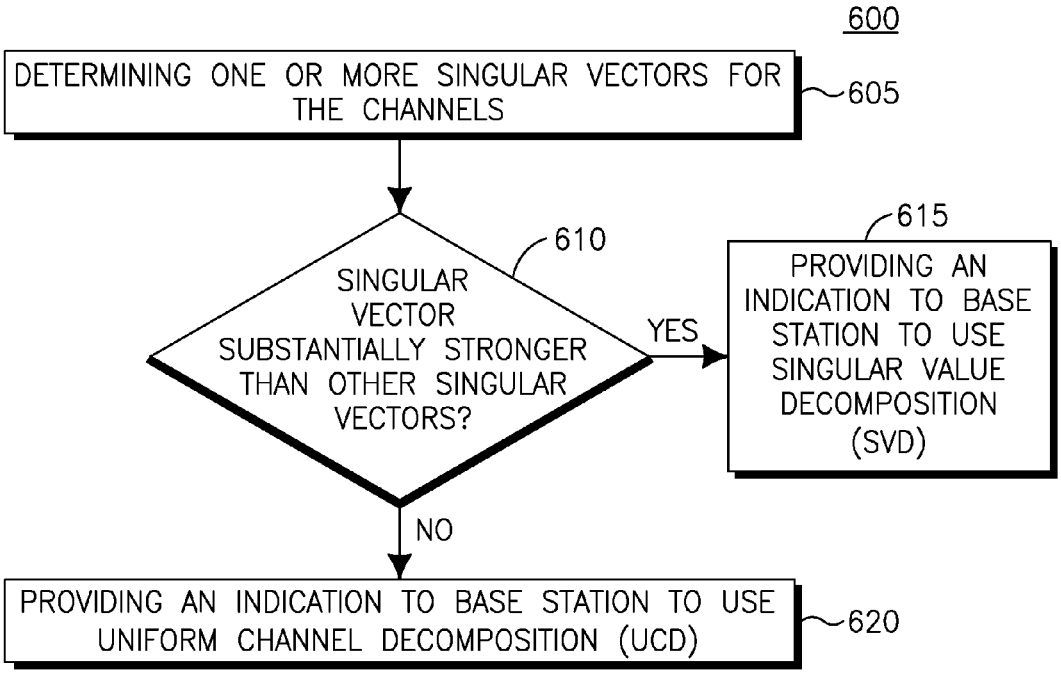
600
DETERMINING ONE OR MORE SINGULAR VECTORS FOR THE CHANNELS
605
610
SINGULAR VECTOR SUBSTANTIALLY STRONGER THAN OTHER SINGULAR VECTORS?
YES
615
PROVIDING AN INDICATION TO BASE STATION TO USE SINGULAR VALUE DECOMPOSITION (SVD)
NO
PROVIDING AN INDICATION TO BASE STATION TO USE UNIFORM CHANNEL DECOMPOSITION (UCD)
620

FIG. 6

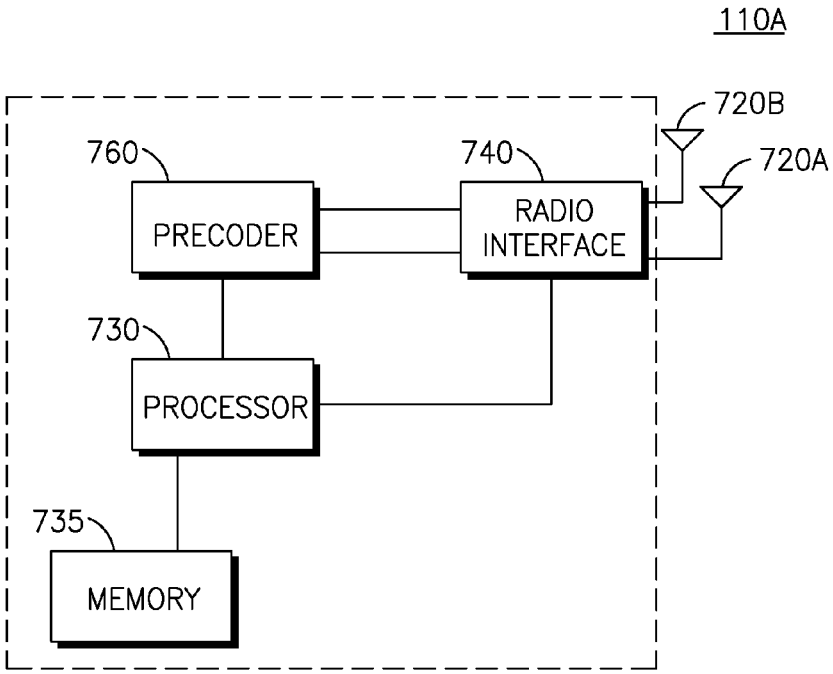
110A
720B
720A
760
PRECODER
740
RADIO INTERFACE
730
PROCESSOR
735
MEMORY

FIG. 7

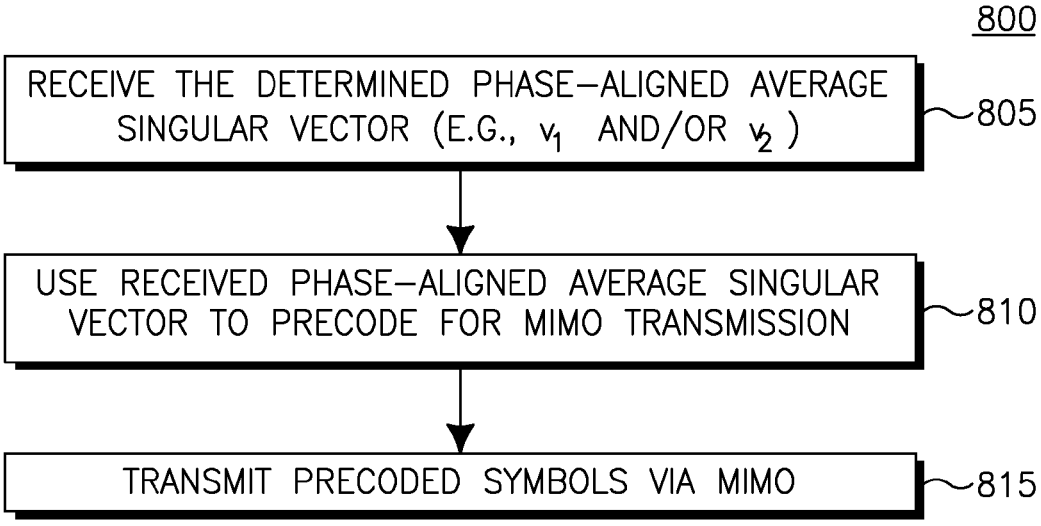
800
RECEIVE THE DETERMINED PHASE-ALIGNED AVERAGE SINGULAR VECTOR (E.G., v_1 AND/OR v_2)
805
USE RECEIVED PHASE-ALIGNED AVERAGE SINGULAR VECTOR TO PRECODE FOR MIMO TRANSMISSION
810
TRANSMIT PRECODED SYMBOLS VIA MIMO
815

FIG. 8

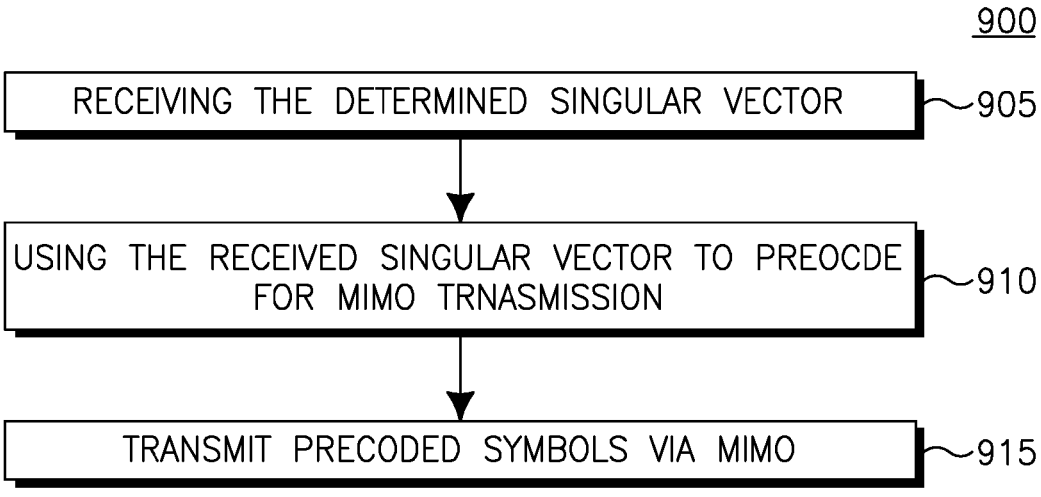
900
RECEIVING THE DETERMINED SINGULAR VECTOR
905
USING THE RECEIVED SINGULAR VECTOR TO PREOCDE FOR MIMO TRNASMISSION
910
TRANSMIT PRECODED SYMBOLS VIA MIMO
915

FIG. 9

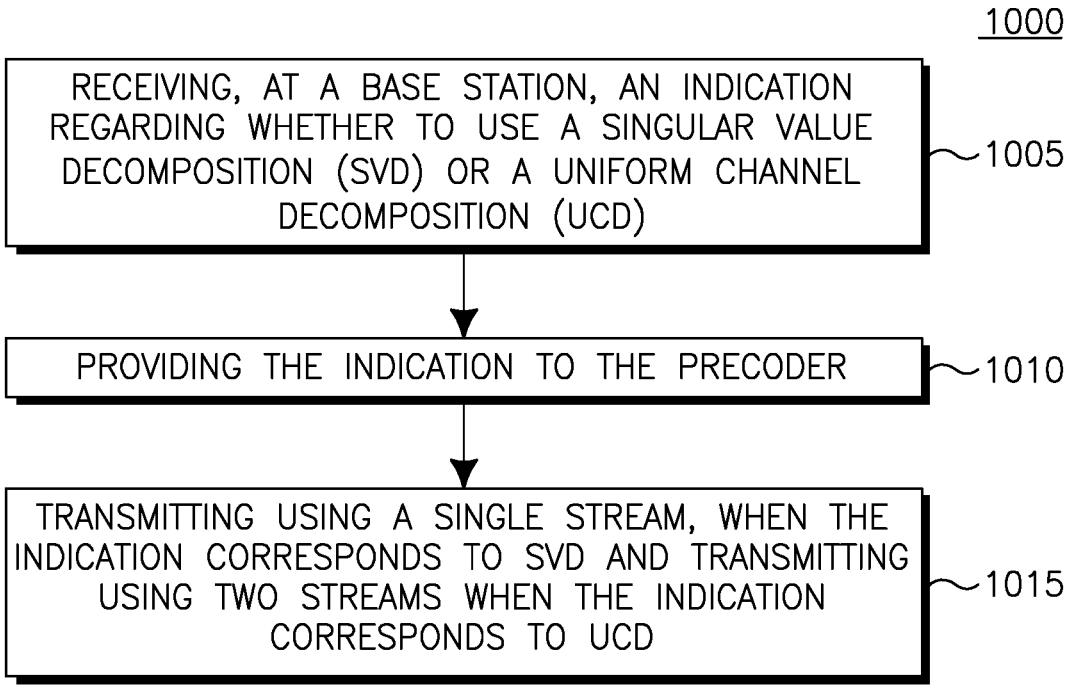
1000
RECEIVING, AT A BASE STATION, AN INDICATION REGARDING WHETHER TO USE A SINGULAR VALUE DECOMPOSITION (SVD) OR A UNIFORM CHANNEL DECOMPOSITION (UCD)
1005
PROVIDING THE INDICATION TO THE PRECODER
1010
TRANSMITTING USING A SINGLE STREAM, WHEN THE INDICATION CORRESPONDS TO SVD AND TRANSMITTING USING TWO STREAMS WHEN THE INDICATION CORRESPONDS TO UCD
1015

FIG. 10

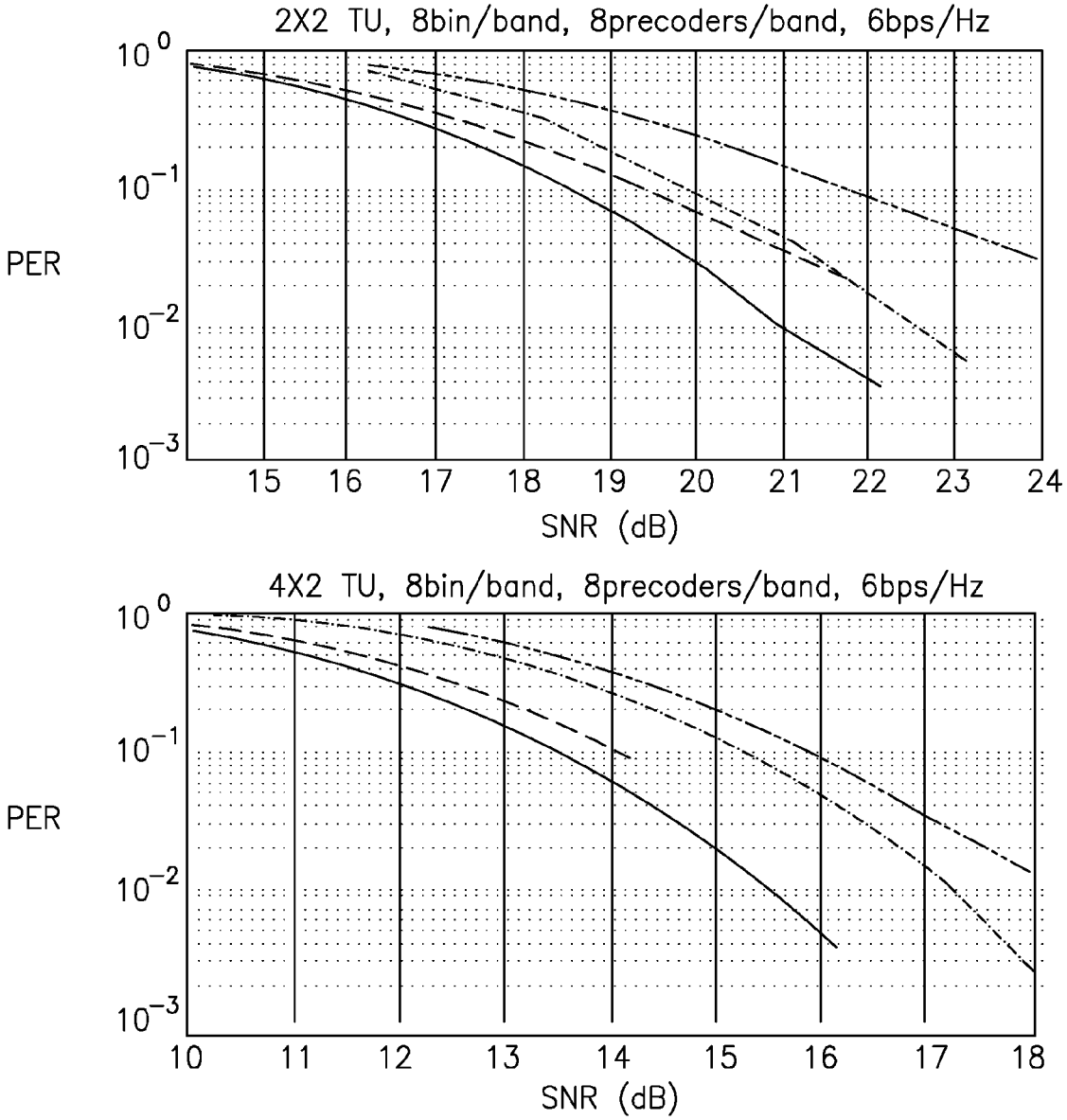
2X2 TU, 8bin/band, 8precoders/band, 6bps/Hz
PER
10^0
10^{-1}
10^{-2}
10^{-3}
15 16 17 18 19 20 21 22 23 24
SNR (dB)
4X2 TU, 8bin/band, 8precoders/band, 6bps/Hz
PER
10^0
10^{-1}
10^{-2}
10^{-3}
10 11 12 13 14 15 16 17 18
SNR (dB)
SVD RxCORR=.5
SVD RxCORR=.8
UCD RxCORR=.5
UCD RxCORR=.8

FIG. 11

CLOSED FORM SINGULAR VALUE DECOMPOSITION

CROSS REFERENCE TO RELATED APPLICATIONS

This application is a continuation of U.S. patent application Ser. No. 12/986,634, filed Jan. 7, 2011, which is a continuation of PCT Application No. PCT/US2009/049851 filed Jul. 7, 2009, which claims the benefit of U.S. Patent Application Ser. No. 61/078,766 filed Jul. 7, 2008. U.S. patent application Ser. No. 12/986,635, filed Jan. 7, 2011, is also a continuation of PCT Application No. PCT/US2009/049852 filed Jul. 7, 2009, which claims the benefit of U.S. Patent Application Ser. No. 61/078,765 filed Jul. 7, 2008. U.S. patent application Ser. No. 12/986,635, filed Jan. 7, 2011, is also a continuation of PCT Application No. PCT/US2009/049853 filed Jul. 7, 2009, which claims the benefit of U.S. Patent Application Ser. No. 61/078,767 filed Jul. 7, 2008. All of the above referenced applications are hereby incorporated by reference herein.

TECHNICAL FIELD

The subject matter described herein relates to wireless communications.

BACKGROUND

In signal processing associated with wireless devices, the singular value decomposition is frequently used to process signals. The singular value decomposition is a factorization of a rectangular real or a complex matrix. For example, the singular value decomposition is a decomposition that may include determining a pseudoinverse, a least squares fitting of data, a matrix approximation, and determining the rank, the range and/or the null space of a matrix. As such, the singular value decomposition is a computationally intensive operation, which in the case of a wireless device, may be problematic. Moreover, in many implementations, the singular value decomposition is an iterative solution, not of a closed form.

Multiple-input and multiple-output (MIMO) is typically used in wireless communications to enhance performance, when compared to non-MIMO approaches. For example, multiple antennas may be implemented at the transmitter and/or the receiver to improve performance by providing, in some implementations, enhanced throughput and range. Often, these performance enhancements may be obtained without substantial increases in transmitted power and/or bandwidth, hence the appeal of MIMO. However, MIMO typically comes at the cost of complex processing, including complex singular value decomposition processing, at the transmitter and at the receiver

SUMMARY

The subject matter disclosed herein provides methods and apparatus for determining a singular value decomposition; providing feedback from a client station to a base station; and closed loop operation of a wireless system implementing multiple input multiple output (MIMO).

A method may include receiving a plurality of signals transmitted at a base station implementing a plurality of antennas configured for a multiple-input and multiple-output transmission. Moreover, one or more singular vectors may be determined using a closed form singular value decomposition. The one or more determined singular vectors may be provided to a precoder at the base station as feedback.

A method may include receiving, at a base station, a singular vector determined, at a client station, using a closed form singular value decomposition. The received singular vector may be used when precoding a plurality of signals for transmission using a multiple-input and multiple-output transmission. Moreover, the precoded plurality of signals may be transmitted as the multiple-input and multiple-output transmission.

A method may align a phase of one or more singular vectors. The singular vectors may each represent one of a plurality of subcarriers of a band. Moreover, an average singular vector may be determined for the plurality of subcarriers of the band, the aligned average singular vector representative of the plurality of subcarriers of the band. The aligned average singular vector may be provided, as feedback, to a precoder at a base station.

A method may include receiving, at a base station, an aligned average singular vector. Moreover, the received aligned average singular vector may be used when precoding a plurality of signals for transmission using a multiple-input and multiple-output transmission. Furthermore, the precoded plurality of signals may be transmitted as the multiple-input and multiple-output transmission.

A method may include determining, at a client station, a plurality of singular vectors for channels used in a multiple-input multiple-output (MIMO) transmission from a base station to a client station. Moreover, the client station may provide a first indication to the base station to use a singular value decomposition, when one of the singular vectors is substantially larger (e.g., stronger) than the other singular vector. Furthermore, the client station may provide a second indication to the base station to use a uniform channel decomposition, when one of the singular vectors is not substantially larger than the other singular vector.

A method may include receiving, at a base station from a client station, a first indication to use a singular value decomposition, if one of the singular vectors is substantially larger than the other singular vector. Moreover, the base station may receive from the client station, a second indication to use a uniform channel decomposition, when one of the singular vectors is not substantially larger than the other singular vector. Furthermore, the base station may be configured for transmission based on at least one of the first indication or the second indication received from the client station.

The details of one or more variations of the subject matter described herein are set forth in the accompanying drawings and the description below. Features and advantages of the subject matter described herein will be apparent from the description and drawings, and from the claims.

BRIEF DESCRIPTION OF THE DRAWINGS

A more detailed understanding may be had from the following description, given by way of example in conjunction with the accompanying drawings wherein:

FIG. **1** depicts a block diagram of a network including client stations and base stations;

FIG. **2** depicts a block diagram of a client station;

FIG. **3** depicts a process for determining a phase-aligned average singular vector for a band;

FIG. **4** depicts another process for determining a phase-aligned average singular vector;

FIG. **5** depicts a process for determining the closed form singular value decomposition;

FIG. **6** depicts a process for determining whether to use a singular value decomposition or a uniform channel decomposition based on the strength of singular vectors;

FIG. **7** depicts a block diagram of a base station;

FIG. **8** depicts a process for using the phase-aligned average singular vector at a base station;

FIG. **9** depicts a process, at a base station, for using the singular vectors determined using closed form singular value decomposition;

FIG. **10** depicts a process for configuring a base station to use a singular value decomposition or a uniform channel decomposition; and

FIG. **11** depicts simulation results.

DETAILED DESCRIPTION

FIG. **1** is a simplified functional block diagram of an embodiment of a wireless communication system **100**. The wireless communication system **100** includes a plurality of base stations **110**A and **110**B, each supporting a corresponding service or coverage area **112**A and **112**B. The base stations are capable of communicating with wireless devices within their coverage areas. For example, the first base station **110**A is capable of wirelessly communicating with a first client station **114**A and a second client station **114**B within the coverage area **112**A. The first client station **114**A is also within the coverage area **112**B and is capable of communicating with the second base station **110**B. In this description, the communication path from the base station to the client station is referred to as a downlink **116**A-B and the communication path from the client station to the base station is referred to as an uplink **116**C.

Although, for simplicity, only two base stations are shown in FIG. **1**, a typical wireless communication system **100** may include a larger number of base stations. The base stations **110**A and **110**B can be configured as cellular base station transceiver subsystems, gateways, access points, radio frequency (RF) repeaters, frame repeaters, nodes, or any wireless network entry point.

The base stations **110**A and **110**B can be configured to support an omni-directional coverage area or a sectored coverage area. For example, the second base station **110**B is depicted as supporting the sectored coverage area **112**B. The coverage area **112**B is depicted as having three sectors, **118**A, **118**B, and **118**C. In typical embodiments, the second base station **110**B treats each sector **118** as effectively a distinct coverage area.

Although only two client stations **114**A and **114**B are shown in the wireless communication system **100**, typical systems are configured to support a large number of client stations. The client stations **114**A and **114**B can be mobile, nomadic, or stationary units. The client stations **114**A and **114**B are often referred to as, for example, mobile stations, mobile units, subscriber stations, wireless terminals, or the like. A client station can be, for example, a wireless handheld device, a vehicle mounted device, a portable device, client premise equipment, a fixed location device, a wireless plug-in accessory or the like. In some cases, a client station can take the form of a handheld computer, notebook computer, wireless telephone, personal digital assistant, wireless email device, personal media player, meter reading equipment or the like and may include a display mechanism, microphone, speaker and memory.

In a typical system, the base stations **110**A and **110**B also communicate with each other and a network control module **124** over backhaul links **122**A and **122**B. The backhaul links **122**A and **122**B may include wired and wireless communication links. The network control module **124** provides network administration and coordination as well as other overhead, coupling, and supervisory functions for the wireless communication system **100**.

In some embodiments, the wireless communication system **100** can be configured to support both bidirectional communication and unidirectional communication. In a bidirectional network, the client station is capable of both receiving information from and providing information to the wireless communications network. Applications operating over the bidirectional communications channel include traditional voice and data applications. In a unidirectional network, the client station is capable of receiving information from the wireless communications network, but may have limited or no ability to provide information to the network. Applications operating over the unidirectional communications channel include broadcast and multicast applications. In one embodiment, the wireless system **100** supports both bidirectional and unidirectional communications. In such an embodiment, the network control module **124** is also coupled to external entities via, for example, content link **126** (e.g., a source of digital video and/or multimedia) and two-way traffic link **128**.

The wireless communication system **100** can be configured to use Orthogonal Frequency Division Multiple Access (OFDMA) communication techniques. For example, the wireless communication system **100** can be configured to substantially comply with a standard system specification, such as IEEE 802.16 and its progeny or some other wireless standard such as, for example, WiBro, WiFi, Long Term Evolution (LTE), or it may be a proprietary system. The subject matter described herein is not limited to application to OFDMA systems or to the noted standards and specifications. The description in the context of an OFDMA system is offered for the purposes of providing a particular example only.

As used herein, IEEE 802.16 refers to one or more Institute of Electrical and Electronic Engineers (IEEE) Standard for Local and metropolitan area networks, Part 16: Air Interface for Fixed Broadband Wireless Access Systems, 1 Oct. 2004, IEEE Standard for Local and metropolitan area networks, Part 16: Air Interface for Fixed and Mobile Broadband Wireless Access Systems, 26 Feb. 2006, and any subsequent additions or revisions to the IEEE 802.16 series of standards.

In some embodiments, downlinks **116**A-B and uplink **116**C each represent a radio frequency (RF) signal. The RF signal may include data, such as voice, video, images, Internet Protocol (IP) packets, control information, and any other type of information. When IEEE-802.16 is used, the RF signal may use OFDMA. OFDMA is a multi-user version of orthogonal frequency division multiplexing (OFDM). In OFDMA, multiple access is achieved by assigning to individual users groups of subcarriers (also referred to as subchannels or tones). The subcarriers are modulated using BPSK (binary phase shift keying), QPSK (quadrature phase shift keying), QAM (quadrature amplitude modulation), and carry symbols (also referred to as OFDMA symbols) including data coded using a forward error-correction code.

In some embodiments, a base station is implemented using multiple-input and multiple-output (MIMO). When MIMO is used, a base station may include a plurality of antennas. For example, base station **110**A may be configured for MIMO and include a precoder (described further below) coupled to two antennas for transmitting the MIMO transmission via downlinks **116**A-B. A client station may include a plurality of antennas to receive the MIMO transmission sent via downlinks **116**A-B. The precoder is configured to perform "precoding," which refers to beamforming to support MIMO

transmission at each of the antennas (e.g., using singular vectors to weight orthogonal "eigen-beams" transmitted via each of the antennas). The client station may also combine the received signals, which may result in fewer errors and/or enhanced data transfer. Although the examples given herein are made in the context of MIMO, other smart antenna techniques may be used as well including MISO (multiple input, single output) and SIMO (single input, multiple output).

Moreover, when MIMO is used, the base station may perform precoding (which may use additional information, such as, for example, channel estimation information) to code, for each antenna, one or more streams of symbols for transmission over the corresponding antenna. In a closed loop feedback-based approach, the channel estimation information may be provided by the client station to the base station. For example, a client station may receive each of the downlinks **116**A-B transmitted by the antennas of the base station, decode the received downlink signals, determine channel estimation information for the decoded channels (e.g., subcarriers) in each of the received downlink signals, and then provide to the base station the determined channel estimation information. The channel estimation information provided by the client station may include singular vectors determined by the client station using a singular value decomposition (SVD). Although the channel estimation information is described as including singular vectors, the channel estimation information may also include other channel information, such as the signal-to-noise ratio of a subcarrier, carrier-to-noise ratio, a channel covariance matrix, a channel matrix, and the like.

To determine the singular vectors, the client station may perform a singular value decomposition (SVD). The client station may provide, as feedback, the singular vectors determined using the singular value decomposition. The singular vectors may be determined in a variety of ways. A channel estimator at the client station may determine the singular vectors v_1 and v_2 (also referred to as u_1 and u_2), as described below. Although the following describes determining singular vectors V_1 and v_2 for the case of an N×2 matrix (i.e., when the client station has 2 antennas), the closed form may be extended to client stations with other quantities of antennas. Further, a typical singular value decomposition is an iterative calculation, which may be computationally intensive for a client station. As such, some or all of the subject matter described herein may relate to a closed form singular value decomposition, which may be less computationally intensive when compared to typical, iterative singular value decomposition algorithms.

Alternatively or additionally, the singular vectors may be determined for each of the channels (e.g., subcarriers) used by the antennas transmitting from the base station to the client station. For example, the base station may include two antennas, each of which transmits over a channel comprising one or more subcarriers. The client station may then determine singular vectors for the subcarriers. Moreover, the subcarriers (as well as other channel information) may be used to determine the so-called "strength" of the channel. For example, in the two antenna case, if the singular vector V_1 is substantially larger (e.g., when the eigen-subchannel of Vi is much larger than eigen-subchannel of V_2) than the singular vector V_2, then the client station may provide an indication to the base station to use a singular value decomposition at the precoder. However, if the singular vector V_1 is not substantially larger than the singular vector v_2, then the client station may provide an indication to the base station to use uniform channel decomposition when precoding at the base station. In some implementations, configuring the base station for a single stream of symbols using a singular value decomposition or a plurality of streams of symbols using a uniform channel decomposition (e.g., based on feedback from the client station indicating the "strength" of the singular vectors) provides enhanced performance, compared to operation using only a singular value decomposition or only a uniform channel decomposition.

FIG. **2** depicts an exemplary client station, such as, for example, client station **114**B. The client station **114**B may include a plurality of antennas **220**A-B for receiving the downlinks **116**A-B, each transmitted by a base station, such as base station **110**A, which may implement MIMO as described further below. Although the examples described herein refer to two antennas at the base station and two antennas at the client station, other quantities of antennas may be used at the base station and the client station. The client station **114**B also includes a radio interface **240**, which may include other components, such as filters, converters (e.g., digital-to-analog converters and the like), symbol demappers, an Inverse Fast Fourier Transform (IFFT) module, and the like, to process the received MIMO transmission sent via downlinks **116**A-B, to determine channel estimation information, and to decode any data, such as the symbols, carried by the downlinks. In some implementations, the client station **114**B is also compatible with IEEE 802.16, OFDMA, and MIMO transmissions (which may be sent via downlinks **116**A-B). MIMO implementations using other wireless technologies, such as LTE, WiBro, and the like, may also be implemented using the subject matter described herein. The client station **114**B may further include a channel estimator **260** (described further below), a processor **220** for controlling client station **114**B and for accessing and executing program code stored in memory **225**.

For each of the MIMO transmissions sent via downlinks **116**A-B and received at each of antennas **220**A-B, the channel estimator **260** may determine channel estimation information, such as singular vectors determined using a singular value decomposition. For example, for each of the subcarriers of a band, the channel estimator **260** may determine singular vectors using a singular value decomposition. In some embodiments, rather than or in addition to performing a traditional, iterative singular value decomposition to determine the singular vectors, the channel estimator **260** may perform a closed form singular value decomposition. The closed form singular value decomposition may be used to determine singular vectors v_1 and v_2. For each of the subcarriers, channel estimator **260** may determine one or more of the singular vectors v_1 and v_2. Moreover, the channel estimator **260** may then provide, based on the strength of the determined singular vectors, an indication to the precoder at the base station to use a singular value decomposition or to use a uniform channel decomposition. When a singular value decomposition is used, the precoder at the base station provides, based on a singular value decomposition, a single stream of symbols for MIMO transmission. When uniform channel decomposition is used, the precoder at the base station provides, based on a uniform channel decomposition, a plurality of streams of symbols for MIMO transmission. The determined singular vectors (as well as any other channel estimation information) may then be provided as feedback by the client station to a precoder at the base station (e.g., as a management message transmitted via uplink **116**C), which may be used at the precoder to configure for a singular value decomposition or a uniform channel decomposition.

In some embodiments, channel estimator **260** may determine the singular vector v_i based on the following:

$$v_1 = h_1^* \cos\theta + h_2^* \sin\theta e^{j\phi},$$

and may determine the singular vector v_2 based on the following:

$$v_2 = h_1^* \sin\theta + h_2^* \cos\theta e^{j\phi}.$$

The determined singular vectors v_1 and v_2 may then be provided as feedback to the base station (e.g., as a management message transmitted via uplink **116**C).

The following provides example implementations for determining the singular vectors v_1 and v_2, although other approaches may be used as well. Generally, analog feedback may provide advantages, such as low complexity at the client station (e.g., a mobile station) and unbounded feedback accuracy which may be particularly important for multi-user MIMO with uncorrelated antennas or multi-base station MIMO.

There are several options for feedback, such as using a full channel matrix H, a covariance matrix $H^H H$, a strongest (e.g., largest) right singular vector, and two strongest right singular vectors. While feeding back the full channel matrix or channel covariance provides sufficient information for single user and/or multi-user MIMO, feeding back just the first or two strongest eigenvectors may reduce feedback overhead.

For example, a 4 antenna base station requires 8 complex values for feedback consisting of a full channel matrix or a covariance matrix, and only 3 complex values for the strongest right singular vector option and 5 complex values for the two strongest right singular vectors option. An 8 antenna base station or two 4 antenna base stations in multi-base station MIMO will require 16, 7, and 13 values for the full channel matrix option, the strongest right singular vector option and the two strongest right singular vectors option, respectively. In addition, a typical operation of closed loop MIMO in frequency division duplex may require the client station (e.g., a mobile station) to send an initial estimate of the post precoding carrier-to-interference-plus-noise ratio (CINR).

To further illustrate, the following provides an example of how the singular values of the channel may be calculated, as these will determine the post precoding signal-to-noise ratio (SNR).

Assume the channel matrix H is of dimension 2 by N (2×N), where N is greater than or equal to 2 (N>=2). The singular values can be found from the eigenvalues of HH^H, which is a 2×2 dimension matrix and which may be solved based on the following two equations:

$$\sigma_1^2 + \sigma_2^2 = Tr(HH^H)$$

$$\sigma_1^2 \sigma_2^2 = det(HH^H),$$

where Tr is trace and σi and σ_2 are the two singular values of H.

To calculate the singular value decomposition, the focus is on the matrix H^H, which may be of size N×2. Then, the correct singular vectors that are of size 2 may be found and those singular vectors may be used to find the left singular vectors that are the right singular vectors of H.

First, the form of H^H may be of the following form: $H^H = V\Sigma U^H$, where U may be generally expressed as follows:

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta e^{j\phi} & -\cos\theta e^{j\phi} \end{pmatrix}.$$

By the definition of the singular value decomposition, the first column of matrix U may be calculated according to the following equation:

$$\theta, \phi = \underset{\theta,\phi}{\operatorname{argmax}} \left\| H^H \begin{pmatrix} \cos\theta \\ \sin\theta e^{j\phi} \end{pmatrix} \right\|,$$

where ||x|| represents a Euclidean norm. The maximum Euclidean norm is the maximal singular value σ_1.

Denoting the i^{th} row of matrix H as h_i and developing the above expression, the following equation is formed:

$$e^{j\phi} = \frac{h_2^* h_1}{|h_2^* h_1|},$$

which may be expressed (e.g., via substitution) as follows:

$$\theta = \underset{\theta}{\operatorname{argmax}} |h_1|^2 \cos^2\theta + |h_2|^2 \sin^2\theta + 2|h_2^* h_1| \sin\theta\cos\theta.$$

Differentiating and equating to zero, the following equation is formed:

$$\tan 2\theta = \frac{2h_2^* h_1}{|h_1|^2 - |h_2|^2},$$

after which a CORDIC rotation is used to calculate cos θ and sin θ. Using $UH^H = V\Sigma$, the strongest singular vector may be determined by normalizing the following equation:

$$v_1 = h_1^* \cos\theta + h_2^* \sin\theta e^{j\phi}.$$

At this point, the strongest right singular vector may be finalized. To finalize the two right strongest singular vectors, a solution to the following equation may be found:

$v_2 = h_1^* \sin\theta + h_2^* \cos\theta e^{j\phi}$, which may be normalized.

To illustrate further, the singular value decomposition (SVD) is generally done through iterative procedures. But for the special case of an N×2 matrix, the decomposition may be done via a closed form solution for singular value decomposition. For example, given a matrix of the following form:

$H = [h_1 h_2] \in C^{N\times 2}$, and then calculating the singular value decomposition as follows:

$$H = U\Sigma V^*,$$

where $U = [u_1 u_2] \in C^{N\times 2}$ is a orthonormal matrix, and Σ is of the following form:

$$\sum = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix},$$

and V is of the following form:

$$V = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta e^{j\phi} & -\cos\theta e^{j\phi} \end{pmatrix}.$$

By the definition of the singular value decomposition, the first column of V, which is parameterized by θ and φ, is calculated according to the following equation:

$$\theta, \phi = \underset{\theta,\phi}{\operatorname{argmax}} \left\| H \begin{pmatrix} \cos\theta \\ \sin\theta e^{j\phi} \end{pmatrix} \right\|,$$

where ||x|| represents the Euclidean norm and this maximum Euclidean norm is the maximal singular value σ_1. Denoting a_i, b_i as the ith element of h_1 and h_2, respectively, the solution of the θ and ϕ equation above may be derived based on the following equation:

$$\theta, \phi = \underset{\theta,\phi}{\operatorname{argmax}} \sum_{i=1}^{N} |a_i \cos\theta + b_i \sin\theta e^{j\phi}|^2 =$$

$$\underset{\theta,\phi}{\operatorname{argmax}} \sum_{i=1}^{N} |a_i|^2 \cos^2\theta + \sum_{i=1}^{N} |b_i|^2 \sin^2\theta + 2Re\left\{ \sum_{i=1}^{N} a_i b_i^* \sin\theta \cos\theta e^{-j\phi} \right\}.$$

Thus, the optimal $\phi = \angle(\Sigma_{i=1}^{N} a_i b_i^*)$, which may be expressed as follows:

$$e^{j\phi} = \frac{\sum_{i=1}^{N} a_i b_i^*}{\left| \sum_{i=1}^{N} a_i b_i^* \right|}.$$

Further simplification of the θ and ϕ equation yields the following:

$$\theta = \underset{\theta}{\operatorname{argmax}} \sum_{i=1}^{N} |a_i|^2 \cos^2\theta + \sum_{i=1}^{N} |b_i|^2 \sin^2\theta + 2\left| \sum_{i=1}^{N} a_i b_i^* \right| \sin\theta\cos\theta,$$

and further constraining

$$\theta \in \left[0, \frac{\pi}{2}\right)$$

causes no loss of optimality, and equating the derivative to zero yields the following:

$$\left(\sum_{i=1}^{N} |b_i|^2 - \sum_{i=1}^{N} |a_i|^2 \right) \sin 2\theta + 2\left| \sum_{i=1}^{N} a_i b_i^* \right| \cos 2\theta = 0.$$

Hence, the following:

$$\tan 2\theta = \frac{2\left| \sum_{i=1}^{N} a_i b_i^* \right|}{\sum_{i=1}^{N} |a_i|^2 - \sum_{i=1}^{N} |b_i|^2}.$$

The value of θ may be calculated based on the above equation, which may have only one solution in the interval 0 to π/2. However, since the solution of interest is cos θ and sin θ rather than θ itself, the following may be used to find a solution:

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}.$$

When α is denoted as follows:

$$\alpha = \frac{\sum_{i=1}^{N} |a_i|^2 - \sum_{i=1}^{N} |b_i|^2}{2\left| \sum_{i=1}^{N} a_i b_i^* \right|}.$$

The following equation results:

$$\frac{1 - \tan^2\theta}{2\tan\theta} = \alpha,$$

which may lead to the unique solution in θ over the interval of 0 to π/2

$$\left(\theta \in \left[0, \frac{\pi}{2}\right)\right),$$

as follows:

$$\tan\theta = \sqrt{1 + \alpha^2} - \alpha;$$

$$\cos\theta = \frac{1}{\sqrt{1 + \tan^2\theta}}; \text{ and}$$

$$\sin\theta = \cos\theta * \tan\theta.$$

Given the above cos θ and sin θ, V may be formed as follows:

$$V = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta e^{j\phi} & -\cos\theta e^{j\phi} \end{pmatrix}$$

and given the following:

$$U\Sigma = HV,$$

and also given the following:

$$u = h_1 \cos\theta + h_2 \sin\theta e^{j\Phi},$$

which may be the first column of UΣ, the largest singular value may have the following form:

$$\sigma_1 = \|u\| = \sqrt{\sum_{i=1}^{N} |a_i|^2 \cos^2\theta + \sum_{i=1}^{N} |b_i|^2 \sin^2\theta + 2\left| \sum_{i=1}^{N} a_i b_i \right| \cos\theta \sin\theta},$$

wherein ||x|| represents the Euclidean norm and the second equality follows from the θ symbol above. As such, the corresponding singular vector is:

$$u_1 = \frac{u}{\sigma_1}.$$

Given the following equation,

$$v=h_1 \sin\theta - h_2 \cos\theta e^{j\Phi},$$

which may be the second column of UΣ, and given $\sigma_1^2+\sigma_2^2=\|H\|_F^2$, where $\|x\|_F$ represents the Frobenius norm, the smaller singular value may be obtained as follows:

$$\sigma_2 = \sqrt{\sum_{i=1}^{N}|a_i|^2 + \sum_{i=1}^{N}|b_i|^2 - \sigma_1^2}\,.$$

Thus, the corresponding singular vector may be:

$$u_2 = \frac{v}{\sigma_2}.$$

Table 1 depicts example Matlab pseudo-code for determining the singular vectors as described above.

TABLE 1

Sample Pseudo-Code

```
function [U, S, V] = SVDN×2(H)
t1 = Σ_{i=1}^{N} |H(i, 1)|^2; %2N real multiplications
t2 = Σ_{i=1}^{N} |H(i, 2)|^2; %2N real multiplications
t3 = Σ_{i=1}^{N} H(i, 1) * conj(H(i, 2)); %4N real multiplications
t4 = abs(t3); %2 real multip. and 1 sqrt
e^{jΦ} = t3/t4; %1 division
t6 = (t1 − t2)/(2 * t4); %1 division
tan θ = sqrt(1+t6^2) − t6; %1 multi. 1 sqrt
cos θ = 1/sqrt(1+tan^2 θ); %1 multi. 1 sqrt. 1 division.
sin θ = tan θ * cos θ; %1 multi.
t5 = t1 * cos^2 θ + t2 * sin^2 θ + 2 * t4^2 * cosθ * sinθ; %7 multi.
σ1 = sqrt(t5); %1 sqrt.
σ2 = sqrt(t1+T2-t5); %1 sqrt.
S = [ σ1   0
      0    σ2 ];
V = [ cosθ          sinθ
      sinθ e^{jφ}   −cosθ e^{jφ} ];
t7 = h2 * e^{jΦ}; %4N real multi.
u1 = (h1 cos θ + t7 sin θ)/σ1; %4N real multi. 1 division
u2 = (h2 sin θ − t7 cos θ)/σ1; %4N real multi. 1 division
U = [u1, u2].
```

Once the singular vectors (e.g., v_1 and v_2, which may also be referred to as singular vectors u_1 and u_2) are determined by channel estimator **260**, channel estimator **260** may provide the singular vectors to a precoder at the base station. For example, channel estimator **260** may provide the determined singular vectors v_1 and v_2 to processor **220**, which forwards the determined singular vectors v_1 and v_2 to the base station in a management message via radio interface **240**, one of the antennas **220**A or **220**B, and uplink **116**C.

Rather than provide singular vectors for each of the subcarriers of a band (which may result in a considerable amount of the uplink **116**C being used for overhead), closed loop MIMO may instead provide, as feedback, one set of singular vectors v_1 and v_2 per band, wherein a typical band is 400-800 KHz and spans 36-72 subcarriers. For example, that one set of singular vectors V_1 and v_2 per band may be determined for a central subcarrier of the band. However, this central subcarrier approach may not accurately represent the band. The subject matter described herein may relate to determining, at a client station, a phase-aligned, average singular vector for the band. For example, to determine the phase-aligned, average singular vector, the client station aligns the phase of each of the singular vectors of the subcarriers of the band, and then determines the average of the singular vector over the subcarriers of the band. The determined phase-aligned, average singular vector may be provided to the base station to configure a precoder for a MIMO transmission via downlinks **116**A-B. In some implementations, the determined phase-aligned, average singular vector more accurately represents the band when compared to using the central subcarrier or using an average across the band that is not phase aligned.

For each of the MIMO transmissions sent via downlinks **116**A-B and received at each of antennas **220**A-B, the channel estimator **260** may determine channel estimation information. For example, for each of the subcarriers of a band, the channel estimator **260** may determine singular vectors using a singular value decomposition. In some embodiments, rather than perform a traditional, singular value decomposition to determine the singular vectors for each of the subcarriers, the channel estimator **260** may perform a closed form singular value decomposition to determine the singular vectors V_1 and v_2, as described above. For each of the subcarriers, channel estimator **260** may determine one or more of the singular vectors v_1 and v_2.

To determine the phase-aligned, average singular vector, channel estimator **260** may select one of the subcarriers and then use that subcarrier's singular vector v_1 to phase align the other subcarrier's singular vectors v_1. Once phase-aligned, the channel estimator **260** may determine the average of the singular vectors v_1 across all of the subcarriers of the band, and then normalize the average. In some embodiments, the channel estimator **260** may repeat the above process using the determined phase-aligned average singular vector v_1 (e.g., rather than selecting one of the subcarriers to align the phases of the other subcarriers, using the determined average and then aligning the phase of all of the other subcarriers relative to the determined average singular vector, and so forth). To determine the phase-aligned average singular vector for singular vector v_2, channel estimator **260** may repeat the above-described process using the singular vector v_2 of each of the subcarriers.

The following describes an exemplary implementation for determining average singular vectors including phase-aligned average singular vector for singular vectors v_1 and v_2.

Specifically, rather than using the central subcarrier of the band, the following four algorithms are described for designing a beamforming vector for a band which contains multiple subcarriers. The algorithms described below may be for one-stream schemes, but they may also be applied to two-stream schemes.

In the first algorithm, let B be the set of subcarrier indices of the band. The first algorithm may include the following:

1. Compute the matrix

$$\sum_{i\in B} H_i H_i^*.$$

2. Obtain the principle eigen-vector v of

$$\sum_{i\in B} H_i H_i^*.$$

The vector v may be the beamforming vector to be used for the whole band.

The second algorithm may include the following:

1. Compute the principle right singular vector v_i of H_i for i∈B.
2. Obtain the matrix

$$\sum_{i\in B} v_i v_i^*.$$

3. Calculate the principle eigen-vector v of

$$\sum_{i\in B} v_i v_i^*,$$

which may be the beamforming vector to be used for the whole band.

The third algorithm (which may include phase alignment) may include the following:

1. Compute the principle right singular vector v_i of H_i for i∈B.
2. Pick up a vector v_j for an arbitrary j∈B and align the phase of other vectors by replacing

$$v_j \leftarrow v_i \frac{v_i^* v_j}{|v_i^* v_j|}$$

so that $v_i^* v_j$ is a positive number.

3. Calculate the norm-1 vector

$$\frac{\sum_{i\in B} v_i}{\left|\sum_{i\in B} v_i\right|},$$

which may be the beamforming vector to be used for the whole band.

The fourth algorithm (which may also include phase alignment) solves the following optimization problem. Generally, the underlying process is to solve the following minimization problem:

$$\min_{v,\phi} \sum \|v_i e^{j\phi_1} - v\|^2,$$

$$\text{s.t. } \|v\| = 1,$$

wherein v is the first or second singular vector of the channel in a subcarrier i. The fourth algorithm may work separately on each singular vector, each of which may not be limited in length. The optimal solution, given known phase ϕ, may be as follows:

$$v = \sum_{i\in B} v_i e^{j\phi_1}.$$

Hence, a solution may be found using the alternate minimization (AM) method as follows: (1) pick any subcarrier j and align the phases of all singular vectors in the band relative to that subcarrier, i.e.,

$$v_j \leftarrow v_i \frac{v_i^* v_j}{|v_i^* v_j|};$$

(2) calculate the average beamforming vector by normalizing the vector; and (3) repeat step 1 by using the vector calculated in step 2. Performing more iterations may generally lead to better performance, but in some cases one or two iterations of steps (1)-(3) may provide sufficient accuracy.

FIG. **3** depicts a process **300** for determining the phase-aligned average singular vector. The description of process **300** will also refer to FIG. **2**. Channel estimator **260** at client station **114**B may be configured to determine the singular vectors for each of the plurality of subcarriers of a band (e.g., using a singular value decomposition). At **305**, channel estimator **260** may align the phase of the singular vectors. At **310**, channel estimator **260** may average the aligned singular vectors across the band, thus yielding the phase-aligned average singular vector for the band. In some embodiments, the channel estimator **260** normalizes the singular vectors before determining the average. For example, the channel estimator **260** may normalize the singular vector v_1 of each of the subcarriers of the band, and then may determine the average of the phase-aligned vectors v_1 of the subcarriers of the band. At **315**, the phase-aligned average singular vector may be provided (e.g., sent) to the base station, for example, to configure a precoder. In some implementations, process **300** is stored as program code and may be stored, for example, at memory **225**.

FIG. **4** depicts a process **400** for determining an aligned average singular vector, which is aligned in accordance with the phase of a reference subcarrier. The description of process **400** will also refer to FIG. **2**. At **405**, channel estimator **260** may compute, based on the channel estimate, a v(i) (i.e., either a singular vector v_1 or a singular vector v_2) for each of the i subcarriers of a multi-subcarrier band. At **410**, channel estimator **260** selects one of the subcarriers as a so-called "reference subcarrier." Channel estimator **260** may align the phase (σ_1) of the singular vectors of the subcarriers in accordance with the phase (σ_i) of the singular vector of the reference subcarrier. At **415**, channel estimator **260** may calculate the average aligned singular vector for the entire multi-subcarrier band from the sum of phase-aligned singular vectors v(i) of all the subcarriers (e.g., all i subcarriers). At **420**, channel estimator **260** may repeat **410** and **415** using the average aligned singular vector determined at **415** as the singular vector of the reference subcarrier. In some implementations, the phase alignment may be performed as described above with respect to algorithms 3 and 4. In some implementations, process **400** is stored as program code and may be stored, for example, at memory **225**.

FIG. **5** depicts a process **500** for determining the closed form singular value decomposition. The description of process **500** will also refer to FIG. **2**. At **505**, a client station may receive a plurality of signals. For example, client station **114**B may receive a plurality of RF signals transmitted by base station **110**B as a multiple-input and multiple-output transmission. At **510**, a singular vector may be determined using a closed form singular value decomposition. For example, channel estimator **260** may determine singular vectors v_1 and v_2 using the closed form singular value decomposition as described herein. At **515**, the determined singular vector may be provided (e.g., sent) to the base station to configure a precoder. In some implementations, process **500** is stored as program code and may be stored, for example, at memory **225**.

FIG. **6** depicts a process **600** for determining, at a client station, whether to use a singular value decomposition or a uniform channel decomposition. The description of process **600** will also refer to FIG. **2**. At **605**, channel estimator **260** may determine one or more singular vectors for the channels (e.g., subcarriers). For example, if base station **110**B transmits using MIMO over two antennas, channel estimator **260** may use a singular value decomposition to determine the singular vectors (e.g., singular vectors v_1 and v_2, also referred to as u_1 and u_2) for each of the channels.

At **610**, channel estimator **260** determines whether one of the singular vectors is substantially larger (e.g., has a higher eigenvalue and is thus capable of more capacity) than the other singular vectors. For example, channel estimator **260** may normalize the determined singular vectors to have a range of zero to one. Given normalized vectors, a substantially larger singular vector v_1 may have a value close to or equal to one, while another singular vector may have a value close to zero. Continuing with this example, at **615**, the channel estimator **260** may provide an indication to base station **110**A to use a singular value decomposition at the precoder. If a singular value decomposition is used at the base station, the precoder at base station **110**B provides, based on singular value decomposition, a single stream of symbols for MIMO transmission. On the other hand, when the singular vector v_1 is not substantially larger than the other singular vector (e.g., singular vector v_1 does not have a value close to or equal to one), at **620**, the channel estimator **260** may provide an indication to the base station to use a uniform channel decomposition at the precoder. In some implementations, process **600** is stored as program code and may be stored, for example, at memory **225**.

FIG. **7** depicts a base station, such as base station **110**A. The base station **110**A includes antennas **720**A-B configured to transmit via downlinks **116**A-B and configured to receive uplink **116**C via at least one of antennas **720**A-B. The base station **110**A may further include a radio interface **740** coupled to the antennas **720**A-B, a precoder **760** (described further below), a processor **730** for controlling base station **110**A and for accessing and executing program code stored in memory **735**. The radio interface **740** may further include other components, such as filters, converters (e.g., digital-to-analog converters and the like), mappers, a Fast Fourier Transform (FFT) module, and the like, to generate symbols for a MIMO transmission via downlinks **116**A-B and to receive the channel estimation information provided via uplink **116**C; to receive the phase-aligned average singular vectors v_1 and v_2 for the band of subcarriers via uplink **116**C; and/or to receive from a client station an indication of whether to use a singular value decomposition or a uniform channel decomposition at the base station when transmitting using MIMO to the client station. The indication may serve as a form of rank adaptation In some implementations, the base station **110**A may also be compatible with IEEE 802.16 and the RF signals of the MIMO downlinks **116**A-B and uplink **116**C are configured in accordance with OFDMA. The precoder **760** may use the determined values of phase-aligned average singular vectors v_1 and v_2 as well as any other channel estimate information (including so-called "side information") about the subcarriers to precode each of the streams to be transmitted as a MIMO transmission by antennas **720**A-B.

The radio interface **740** may decode the uplink **116**C carrying the singular vectors v_1 and v_2 and then provides the singular vectors v_1 and v_2 to the precoder **760**. Alternatively or additionally, the radio interface **740** may decode the uplink **116**C carrying the phase-aligned average singular vectors v_1 and v_2 and then provides the phase-aligned average singular vectors v_1 and v_2 to the precoder **760**. The precoder **760** may use the singular vectors v_1 and v_2 and/or the phase-aligned average singular vectors v_1 and v_2 (as well as any other channel estimation information and/or side information provided as feedback by the client station to the base station) to precode symbols for MIMO transmission via each antenna **720**A-B and downlinks **116**A-B. The term "precoding" refers to beamforming to support MIMO transmission at each of the antennas (e.g., using singular vectors to weight orthogonal "eigen-beams" transmitted via each of the antennas).

Alternatively or additionally, the radio interface **740** may decode the uplink **116**C carrying an indication (e.g., a management message received from a client station) representative of whether the precoder is configured to perform a singular value decomposition or a uniform channel decomposition. The radio interface **740** may also decode uplink **116**C carrying any channel estimation information (e.g., singular vectors determined at the client station), which are provided to the precoder **760**. The precoder **760** receives the indication and may configure for a singular value decomposition or a uniform channel decomposition. When a singular value decomposition is used, the precoder at the base station may provide a single stream for MIMO transmission via the antennas **720**A-B. When uniform channel decomposition is used, the precoder **760** uses any singular vectors v_1 and v_2 determined at the client station (as well as any other channel estimation information provided as feedback by the client station to the base station) to provide, based on a uniform channel decomposition, a plurality of symbols streams (e.g., two symbol streams) for MIMO transmission via each antenna **720**A-B.

FIG. **8** depicts a process **800**, at the base station, for using the phase-aligned average singular vectors determined at the client station. The description of process **800** will refer to FIGS. **2** and **7** as well. At **805**, a base station, such as base station **110**A, receives the phase-aligned average singular vector (e.g., for v_1 and/or v_2) determined at a client station, such as client station **114**B. At **810**, the received phase-aligned average singular vector may be used at a precoder, such as precoder **760**, when precoding signals for transmission using multiple-input and multiple-output transmission. At **815**, the base station may transmit, as the multiple-input and multiple-output transmission, the precoded plurality of signals (e.g., symbols carried via downlink **116**A-B). In some implementations, process **800** is stored as program code and may be stored, for example, at memory **735**.

FIG. **9** depicts a process **900**, at the base station, for using the singular vectors determined using the closed form singular value decomposition. The description of process **900** will refer to FIGS. **2** and **7** as well. At **905**, a base station, such as base station **110**A, receives one or more singular vectors determined at a client station, such as client station **114**B, using the closed form singular value decomposition described herein. At **910**, the received singular vectors may be used at a precoder, such as precoder **760**, when precoding signals for transmission using multiple-input and multiple-output transmission. At **915**, the base station may transmit, as the multiple-input and multiple-output transmission, the precoded plurality of signals (e.g., symbols carried via downlink **116**A-B). In some implementations, process **900** is stored as program code and may be stored, for example, at memory **735**.

Moreover, in some implementations, the client station may feed back the largest singular vector of the collaborating base station antennas. This is referred to as joint antenna processing multi-basestation MIMO in 802.16m or coordinated multi-point transmission (COMP) in LTE. For example, if

each base station has 4 antennas and 3 base stations are collaborating, the singular vector has L=12 complex numbers. The computation of the singular vector(s) for a client station with 2 antennas may be performed as noted above. For example, the largest singular vector for any subcarrier may be given by $v=h_1*\cos\theta+h_2*\sin\theta e^{j\Phi}$, where h_i i=1,2 is the ith row of H, and the 2×L global downlink channel between the client station and all of the collaborating base stations is as follows:

$$e^{j\phi}=\frac{h_2^*h_1}{|h_2^*h_1|} \text{ and } \tan 2\theta=\frac{2|h_2^*h_1|}{|h_1|^2-|h_2|^2}.$$

The phase-aligned singular vectors of several subcarriers may be averaged within a band to get the average singular vector. The resulting singular vector is

$$v=\sum_{k\in S} v_k \frac{v_k^* v_j}{|v_k^* v_j|},$$

where j denotes one of the subcarriers.

FIG. **10** depicts a process **1000** for configuring a base station to use a singular value decomposition adaptation or a uniform channel decomposition based on the strength singular vectors determined by a client station. The description of process **1000** will refer to FIGS. **2** and **7** as well. At **1005**, a base station, such as base station **110**A, receives an indication from a client station representative of whether to use a singular value decomposition or a uniform channel decomposition, when transmitting to the client station. For example, client station **114**B may provide the indication as a management message via uplink **116**C. At **1010**, the received indication may be provided to a precoder, such as precoder **760**. If the indication corresponds to a singular value decomposition, the precoder **760** provides, based on a singular value decomposition, a single stream of symbols for MIMO transmission via antennas **720**A-B. When the indication corresponds to uniform channel decomposition, the precoder **760** may use any singular vectors v_1 and v_2 determined at the client station (as well as any other channel estimation information provided as feedback by the client station to the base station) to provide, based on a uniform channel decomposition, a plurality of symbols streams (e.g., two symbol streams) for MIMO transmission via antennas **720**A-B. At **1015**, the base station may transmit to the client station based on the indication provided by the client station. For example, the base station **110**A may configure transmission to the client station **114**B based on whether the client station **114**B has indicated that the base station **110**A should transmit in accordance with a singular value decomposition or a uniform channel decomposition. In the case of a uniform channel decomposition, the client station **114**B may also provide channel estimation information, such as singular vectors, a channel covariance matrix, and the like. In some implementations, process **1000** is stored as program code and may be stored, for example, at memory **735**.

Moreover, typically, rank-2 operation decomposes the channel using singular value decomposition and transmits on the two eigen-subchannels. This orthogonalization of the channel may cause reduced performance in some implementations, which do not use matched modulation and coding schemes to each singular vector. To address that issue, the eigen-subchannels may be rotated mathematically such that the two subchannels become two layers with identical output SINR when decoded using a successive interference cancellation (SIC) receiver. This is the underlying essence of uniform channel decomposition.

To further illustrate, rank adaptation may be performed using uniform channel decomposition in a practical closed loop MIMO operation. The client station (e.g., mobile station) or base station may decide on the best rank for the transmission according to the channel, SINR, and other considerations using, for example, a capacity criterion. In the case of rank-1 transmission, the strongest singular vector may be used. In the case of rank-2 transmission, the precoder is calculated based on the uniform channel decomposition. In order to calculate the uniform channel decomposition, information regarding the channel's right singular vectors and singular values are required. This is best facilitated in frequency division duplex by the client station feeding back analog feedback of the channel or channel covariance matrix. Alternatively or additionally, the two singular vectors and the ratio of the singular values may also be sent as feedback. Moreover, the feedback may be facilitated by using codebooks and feeding back for rank-2 one extra value representing the ratio of the singular values.

FIG. **11** depicts plots of simulation results showing a comparison of rank-2 transmission between the uniform channel decomposition method and a regular singular value decomposition in 2×2 and 2×4 configurations using a GSM TU channel, wherein one precoder per 9 subcarriers (bin) is used and 6 bits per second per hertz is the combined two stream spectral efficiency. FIG. **11** depicts the gain of uniform channel decomposition approach.

The subject matter described herein may be embodied in systems, apparatus, methods, and/or articles depending on the desired configuration. Base station **110**A (or one or more components therein) can be implemented using one or more of the following: a processor executing program code, an application-specific integrated circuit (ASIC), a digital signal processor (DSP), an embedded processor, a field programmable gate array (FPGA), and/or combinations thereof. Client station **114**B (or one or more components therein) can be implemented using one or more of the following: a processor executing program code, an application-specific integrated circuit (ASIC), a digital signal processor (DSP), an embedded processor, a field programmable gate array (FPGA), and/or combinations thereof. These various implementations may include implementation in one or more computer programs that are executable and/or interpretable on a programmable system including at least one programmable processor, which may be special or general purpose, coupled to receive data and instructions from, and to transmit data and instructions to, a storage system, at least one input device, and at least one output device. These computer programs (also known as programs, software, software applications, applications, components, program code, or code) include machine instructions for a programmable processor, and may be implemented in a high-level procedural and/or object-oriented programming language, and/or in assembly/machine language. As used herein, the term "machine-readable medium" refers to any computer program product, computer-readable medium, apparatus and/or device (e.g., magnetic discs, optical disks, memory, Programmable Logic Devices (PLDs)) used to provide machine instructions and/or data to a programmable processor, including a machine-readable medium that receives machine instructions as a machine-readable signal. Similarly, systems are also described herein that may include a processor and a memory coupled to the processor. The

memory may include one or more programs that cause the processor to perform one or more of the operations described herein.

Although a few variations have been described in detail above, other modifications or additions are possible. In particular, further features and/or variations may be provided in addition to those set forth herein. For example, the above closed form singular value decomposition may be used in other applications, such as at the base station to determine singular vectors for a sounder transmitted by the client station to the base station. Moreover, the implementations described above may be directed to various combinations and subcombinations of the disclosed features and/or combinations and subcombinations of several further features disclosed above. In addition, the logic flow depicted in the accompanying figures and/or described herein does not require the particular order shown, or sequential order, to achieve desirable results. Other embodiments may be within the scope of the following claims.

What is claimed is:

1. A method for use in a client station, the method comprising:

receiving a plurality of signals transmitted by a base station implementing a plurality of antennas configured for a multiple-input and multiple-output transmission;

determining, using a closed form singular value decomposition, one or more singular vectors;

aligning a phase of each of the one or more singular vectors to generate one or more aligned singular vectors;

averaging the aligned singular vectors to generate an average singular vector; and

providing, as feedback, the average singular vector to a precoder at the base station.

2. The method of claim **1**, wherein determining further comprises:

determining, using the closed form singular value decomposition without iteration, the one or more singular vectors.

3. The method of claim **1**, wherein determining further comprises:

determining, using the closed form singular value decomposition, a first singular vector based on the following equation:

$$v_1 = h_1 * \cos\theta + h_2 * \sin\theta e^{j\Phi}; \text{ and}$$

a second singular vector based on the following equation:

$$v_2 = h_1 * \sin\theta + h_2 * \cos\theta e^{j\Phi}.$$

4. A client station comprising:

a radio interface configured to receive a plurality of signals transmitted by a base station implementing a plurality of antennas configured for a multiple-input and multiple-output transmission; and

a channel estimator configured to determine, using a closed form singular value decomposition, one or more singular vectors;

the channel estimator further configured to align a phase of each of the one or more singular vectors to generate one or more aligned singular vectors, and to average the aligned singular vectors to generate an average singular vector;

wherein the radio interface is further configured to transmit, as feedback, the average singular vector to a precoder at the base station.

5. The client station of claim **4**, wherein the channel estimator is further configured to determine, using the closed form singular value decomposition without iteration, the one or more singular vectors.

6. The client station of claim **4**, wherein the channel estimator is further configured to determine, using the closed form singular value decomposition, a first singular vector based on the following equation:

$$v_1 = h_1 * \cos\theta + h_2 * \sin\theta e^{j\Phi}; \text{ and}$$

a second singular vector based on the following equation:

$$v_2 = h_1 * \sin\theta + h_2 * \cos\theta e^{j\Phi}.$$

7. A method for use in a base station, the method comprising:

receiving, at a base station, feedback comprising an average singular vector determined, at a client station, by using a closed form singular value decomposition to determine one or more singular vectors, aligning a phase of each of one or more singular vectors to generate one or more aligned singular vectors, and averaging the one or more aligned singular vectors;

precoding a plurality of signals for multiple-input and multiple-output transmission based on the received average singular vector; and

transmitting, as the multiple-input and multiple-output transmission, the precoded plurality of signals.

8. The method of claim **7**, wherein the one or more singular vectors are determined at the client station using the closed form singular value decomposition without iteration.

9. The method of claim **7**, wherein the one or more singular vectors are determined using the closed form singular value decomposition at the client station based on a first singular vector determined according to the following equation:

$$v_1 = h_1 * \cos\theta + h_2 * \sin\theta e^{j\Phi}; \text{ and}$$

based on a second singular vector determined according to the following equation:

$$v_2 = h_1 * \sin\theta + h_2 * \cos\theta e^{j\Phi}.$$

10. The method of claim **9**, wherein v_1 and v_2 are phase aligned.

11. The method of claim **7**, wherein the precoding the plurality of signals for multiple-input multiple-output transmission includes using the singular vectors to weight orthogonal eigen-beams transmitted via each of a plurality of antennas.

12. A base station comprising:

a radio interface configured to receive feedback comprising an average singular vector determined, at a client station, by using a closed form singular value decomposition to determine one or more singular vectors, aligning a phase of each of one or more singular vectors to generate one or more aligned singular vectors, and averaging the aligned singular vectors; and

a precoder configured to precode a plurality of signals for multiple-input and multiple-output transmission based on the received average singular vector;

wherein the radio interface is further configured to transmit, as the multiple-input and multiple-output transmission, the precoded plurality of signals.

13. The base station of claim **12**, wherein the one or more singular vectors are determined at the client station using the closed form singular value decomposition without iteration.

14. The base station of claim **12**, wherein the one or more singular vectors are determined using the closed form singu-

lar value decomposition at the client station based on a first singular vector determined according to the following equation:

$$v_1 = h_1 * \cos\theta + h_2 * \sin\theta e^{j\Phi}; \text{ and}$$

based on a second singular vector determined according to the following equation:

$$v_2 = h_1 * \sin\theta + h_2 * \cos\theta e^{j\Phi}.$$

15. The base station of claim **14**, wherein v_1 and v_2 are phase aligned.

16. The base station of claim **13**, wherein the precoder is further configured to precode the plurality of signals for multiple-input multiple-output transmission using the average singular vector to weight orthogonal eigen-beams transmitted via each of a plurality of antennas.

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